

Box-Jenkins Models for Inflation, Output Growth, and Impulse Variables

E. J. Bomhoff

The theoretical specification of the price and output relations requires variables for the expected and unexpected parts of the impulse measures and some other time series. This appendix describes the construction of the empirical counterparts to these concepts. The unexpected parts are the residuals ϵ_t of simple Box-Jenkins models (Box and Jenkins, 1970). Normally such models are fitted to the original series, if it is stationary, or to a differenced series if it is not. Here we always took first differences of the series to be modeled, also in those cases where the original series x_t was already stationary. This was done in order to get a series with a mean value sufficiently close to zero to be neglected. Modeling Δx_t means trying to improve upon the naive model: no change between last year and this year in x (an impulse, the growth rate of world money, or some other series). Modeling x_t on the other hand, would have implied that we build upon the naive formula: next year x_t will be equal to its average value over the complete period. It is not attractive to assume knowledge about the mean value of the x_t when the period over which the averaging is done includes years that lie in the future. We avoid this by modeling the Δx_t -series.

There were two reasons for limiting ourselves to the simplest Box-Jenkins models, one statistical and one economic. The statistical reason is the short length of the series: at most twenty-five observations. Equally important is that the more previous values of Δx_t or ϵ_t are included in the information set used for the forecasts, the less acceptable it becomes that no other relevant series is included in the information set. If forecasts based on the adaptive expectations model or on other simple Box-Jenkins models are judged to be too coarse, then current or immediate past values of related variables seem stronger candidates for inclusion in the information set than values from the more distant past of the forecast series itself. (See McCallum, 1976, who also finds that the optimal formula lies somewhere between the simple adaptive expectations model and the type of variable that would be constructed in the first stage of a TSLs regression.)

We have not attempted to fit parameters of an order higher than 2. Of the remaining AR(1), AR(2), MA(1), MA(2), and ARMA(1,1) models, the one with the highest \bar{R}^2 was selected. In Table C1, summary statistics of the models

are presented. The unadjusted value of R^2 is given, because it indicates the degree to which the residuals are different from the Δx_t -series itself.

In some cases models could only be fitted for a slightly different period due to outliers in the final two years (1974, 1975) of the sample. All observations more than 2.5 standard deviations away from the mean were deleted and replaced by values calculated from the model fitted to the shortened series. If the outliers had not been removed, they would have dominated the parameter values too much.

With the Box-Jenkins residuals ϵ_t as the unforeseen parts of a variable, the foreseen value of the variable is found by subtraction.

Table C1

Box-Jenkins Models

Variable	Model	R ²
\hat{p}_c (t-stat.)	$\Delta \hat{p}_{c,t} = (1 - 0.64B) \epsilon_t$ (0.17)	0.27
\hat{p}_i^*	$(1 + 0.29B + 0.60B^2) \Delta \hat{p}_{i,t}^* = \epsilon_{i,t}$ (0.17) (0.17)	.24
\hat{p}_c^*	$\Delta \hat{p}_{c,t}^* = (1 - 0.08B^2) \epsilon_{c,t}$ (0.19)	.01
\hat{y}	$\Delta \hat{y}_t = (1 - 0.50B - 0.43B^2) \epsilon_t$ (0.19) (0.18)	.24
\hat{gdp}^*	$\Delta \hat{gdp}_t^* = (1 - 0.79B) \epsilon_t$ (0.12)	.32
\hat{m}^*	$\Delta \hat{m}_t^* = (1 - 0.33B - 0.53B^2) \epsilon_t$ (0.18) (0.17)	.24
\hat{M}	$\Delta \hat{M}_t = (1 - 0.22B - 0.43B^2) \epsilon_t$ (0.20) (0.20)	.15
\hat{M}^*	$(1 - 0.79B) \Delta \hat{M}_t^* = (1 - 0.97B) \epsilon_t$ (0.27) (0.13)	.08
\hat{S}	$\Delta \hat{S}_t = (1 - 0.89B) \epsilon_t$ (0.10)	.44
FI	$\Delta FI_t = (1 - 0.64B) \epsilon_t$ (0.16)	.16
ΔTP	$\Delta^2 TP_t = (1 - 0.99B) \epsilon_t$ (0.058)	0.47

Note: B is the backshift operator, i.e., $Bx_t \equiv x_{t-1}$.

Means have been neglected (see text).

No observations were lost at the beginning of the series because an estimation procedure was used which included the so-called back-forecasting feature.

Standard deviations, in brackets below each coefficient, are based on a linearization in the vicinity of the optimum and should be regarded as approximative.

All models fitted to 1953-75, except:

\hat{p}_i^* -- 1953-73
 \hat{M} -- 1953-74
 \hat{M}^* -- 1953-73.

Appendix D
Table D1

Additional Regression Equations

	VARU	R^2	DW
16'	0.92	0.87	1.80
$\hat{p}_c = -1.57 + .59 \hat{M}_a^{-.5} + .57 \hat{p}_{c-.5} + 2.43 \text{ shift } 68/69 + 1.29 \Delta^2 TP_{-5} + .10 \Delta m_{na}$			
			(2.02) (4.70) (6.23) (3.24) (3.17) (1.38)
22'	1.61	.78	1.56
$\hat{p}_c = -1.47 + .82 \hat{M}_a^{-.5} + .37 \hat{p}_{i-.5} + 2.46 \text{ shift } 68/69 + 1.26 \Delta^2 TP_{na}^{-.5} + .09 \Delta m_{na}$			(1.42) (5.60) (3.91) (2.46) (2.33) (.92)
			(.30) (5.14) (5.97) (.31) (5.46) (5.45)
5'	0.49	.93	2.29
$\hat{p}_c = .30 + .25 \hat{M}_a^{-1.5} + .70 \hat{p}_{c-1.5} + .04 e_{-1.5} + 3.10 \text{ shift } 68/69 + 1.48 \Delta^2 TP_{-5}$			
			(.71) (5.14) (5.97) (.31) (5.46) (5.45)
			(.14) $\Delta m_{na}^{-.10} - .10 \Delta e$
			(3.30)
11'	0.72	.90	1.90
$\hat{p}_c = .49 + .50 \hat{M}_a^{-1.5} + .42 \hat{p}_{i-1.5} + 3.61 \text{ shift } 68/69 + 1.18 \Delta^2 TP_{-5}$			
			(1.09) (9.31) (9.36) (5.16) (5.16) (3.70)
			(.22) $\Delta m_{na}^{-.10} + .13 \Delta e$
			(3.98)
			(.51) (3.70) (9.44) (6.22) (4.88) (5.33)
16''	0.53	.93	1.84
$\hat{p}_c = .25 + .32 \hat{M}_a^{-.5} + .68 \hat{p}_{c-.5} + .01 e_{-1.5} + 2.86 \text{ shift } 68/69 + 1.39 \Delta^2 TP_{-5}$			
			(.51) (3.70) (9.44) (6.22) (4.88) (5.33)
			(.16) $\Delta m_{na}^{-.10} \Delta e$
			(2.79) (1.93)
22''	1.47	0.82	2.26
$\hat{p}_c = .28 + .58 \hat{M}_a^{-.5} + .52 \hat{p}_{i-.5} + 3.48 \text{ shift } 68/69 + 1.59 \Delta^2 TP_{-5} + .24 \Delta m_{na}$			
			(.36) (4.92) (6.13) (3.56) (3.70) (2.61)
			(.04) Δe
			(.51)

Table D1 -- Continued

28'	$u_{27} = .28 + 1.01 \Delta e$	(.38) (4.38)							
29'	$\hat{p}_i^* = .51 + .91 (\hat{p}_i^* - u_{27}) + 1.03 \Delta e$	(.61) (6.11) (4.37)							
30'	$\hat{p}_i^* = -5.32 + 1.19 \hat{M}_{1.5}^* + 1.03 \Delta e$	(3.54) (6.11) (4.37)							
43'	$\Delta \hat{y}_i = -.61 + .31 \Delta \hat{M}_{1.1} + .60 \Delta \hat{m}_i^* + .59 \Delta (\hat{p}_i^* + e) + .42 \Delta \delta_{-1}$	(1.40) (1.87) (4.53) (2.92) (3.24)							
49'	$\Delta \hat{y}_{ua} = -.57 + .14 \Delta \hat{M}_{ua}^{-1} + .69 \Delta \hat{m}_{ua}^* + .59 (\Delta \hat{p}_{ua}^c + \Delta e) + .38 \Delta \delta_{ua}^{-1}$	(1.80) (.98) (5.51) (2.47) (2.78)							
			2.12	0.84	2.32				
			4.28	.76	2.02				
			12.11	.75	1.75				
			12.11	.75	1.75				
			11.74	0.47	1.72				

Note: Generating the residuals of an ARIMA process fitted on Δe in order to obtain Δe_{ua} and e_{ua} for use in equations 16', 22', and 49' seems unnecessary for the fixed-rate period up to 1972 when all exchange rate fluctuations may be assumed to be unanticipated, and impossible for the managed-floating period since 1972 for lack of sufficient annual observations.

Appendix B
Table E1
Data Series

	\hat{p}_c	\hat{y}	\hat{p}_i	\hat{p}_c^*	\hat{m}^*	FI	-e	\hat{gdp}^*
1952	0.0100	0.0000	-1.2376	4.0518	2.0783	1.9930	0.2101	3.4556
1953	-0.3807	9.7256	-10.5138	0.3663	3.2467	5.9905	0.1049	5.3247
1954	4.5069	10.5393	-2.6241	0.8989	9.1667	7.4392	0.1048	1.6542
1955	2.2348	6.4374	1.6267	0.6032	11.6004	8.2672	-0.5249	8.1884
1956	2.6837	6.0737	3.918	2.4492	7.1390	3.2536	-0.5277	4.7675
1957	5.1453	4.3063	4.6311	2.7849	6.5788	3.6888	0.2114	3.6264
1958	1.5480	0.0000	-5.6253	2.5180	1.3903	1.7004	0.9459	0.2018
1959	1.2324	10.6740	-3.0975	0.2656	9.1667	0.1499	0.9459	6.5497
1960	2.4400	9.6437	-0.2904	0.6509	13.1905	2.9322	0.1042	5.3190
1961	2.3717	3.4016	-1.8673	1.2225	5.2592	2.8296	3.7817	4.9780
1962	2.5863	5.4118	-0.9344	2.4078	8.5260	3.4025	0.7992	5.4403
1963	3.6814	5.1164	1.3311	2.5785	7.8811	2.6272	0.0000	4.8190
1964	6.5788	9.5406	2.3033	2.5375	10.1654	9.1424	-0.0996	6.2843
1965	3.9028	5.3129	0.5187	2.0802	8.9841	3.8877	0.1990	5.2924
1966	5.2877	5.8590	0.7174	3.0781	6.2975	2.7628	-0.5982	5.5075
1967	2.9267	3.9746	-0.7831	2.8490	3.0529	3.6802	0.4988	3.6987
1968	2.5278	11.0801	-2.8502	2.4293	12.8393	4.1064	-0.4988	5.2811
1969	6.0813	11.2397	3.3531	4.4026	13.9762	2.0431	-0.1001	4.6120
1970	4.4017	8.9597	6.2505	5.6758	9.8034	3.6989	0.2000	3.2409
1971	7.7516	5.8269	4.2964	6.0568	7.5107	4.5030	3.5332	4.2149
1972	8.0566	4.6091	-0.6723	8.7251	9.2579	2.1136	8.4114	5.8533
1973	8.6453	6.1154	7.1669	13.3289	10.9751	3.7409	14.3455	5.6012
1974	9.4856	2.5106	29.9882	11.2176	3.9221	3.7889	3.4720	-1.0404
1975	9.9936	-5.0858	4.0662	11.5897	-3.0459	5.7572	6.2543	-0.8859

Mean Values over the period 1954-75

4.7305	5.9795	2.3135	4.1067	7.8927	3.8871	1.8557	4.2366
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Sources: Central Planning Bureau; The Netherlands Bank; Ministry of Finance; Central Bureau of Statistics; IMF International Financial Statistics; Michele Frattanni, EC, Brussels.

	M	M*	S	Y _a	P _a ¹	P _a ^c	H _a	gdp ^a
1952	6.7045	3.5078	8.3695	0.0000	-1.2376	4.0518	1.9930	3.4556
1953	7.9253	3.2680	2.5789	4.8516	-6.0888	4.3563	3.5465	4.5617
1954	6.5981	4.7351	1.6351	9.9893	-2.9811	0.6959	4.5882	4.5302
1955	6.2938	3.8596	4.8725	7.9604	-3.5762	1.5272	5.7772	3.7484
1956	2.7739	3.1789	2.0821	6.7377	-3.5762	0.9159	6.8356	4.4755
1957	-2.9464	1.6110	2.8143	6.8343	1.0931	2.8419	5.7118	4.3464
1958	4.0395	4.1065	10.1997	5.6250	3.9697	2.8496	5.1484	4.0098
1959	9.2441	2.1605	0.7594	3.6540	-2.5875	2.8496	5.1484	4.0098
1960	4.9660	3.4237	4.7522	9.3707	3.0346	0.6299	2.8262	3.5710
1961	7.5043	6.3866	2.4134	6.2846	-1.8683	1.1735	3.0276	3.7430
1962	5.5578	5.8514	2.7602	4.4958	-2.3214	1.5488	3.8093	3.8093
1963	9.2497	8.2142	6.0379	5.9694	0.4891	2.7315	3.1205	3.9540
1964	8.1977	5.7023	8.2647	4.9276	0.8643	2.8415	3.2764	3.9403
1965	10.2528	5.9165	6.8704	7.3929	1.4227	2.8772	5.5487	4.2394
1966	7.0317	4.6874	7.0491	7.520	1.2214	2.4311	5.1158	4.2675
1967	6.7465	9.0736	9.0978	5.6796	2.4809	3.4660	4.4347	4.3347
1968	8.2346	6.8826	6.8111	3.8701	0.2968	3.1283	4.2664	4.0151
1969	9.1464	5.5709	7.8594	8.0067	-0.5889	2.8036	4.4111	4.0870
1970	9.9587	8.9923	7.5997	6.3287	3.5165	4.7828	3.7239	4.0069
1971	15.3526	12.6844	7.4770	6.0489	2.4524	5.8828	3.8780	3.6599
1972	16.2331	18.3205	6.6209	4.5911	3.8997	6.3171	4.2666	3.5853
1973	7.7911	16.1506	2.0812	4.4744	2.7139	9.0399	3.6569	3.8652
1974	2.5622	4.9095	10.5296	5.0716	8.5792	13.4756	3.8499	4.0340
1975	16.8553	11.8343	10.3775	2.8592	19.3462	11.2227	3.9922	2.7991
Mean Values over the Period 1954-75								
7.8020	7.0570	5.8621	5.9239	1.7947	3.9173	4.3170	3.9122	

Table E1 -- Continued

Table E1 -- Continued

	\hat{m}^{*a}	\hat{M}^a	\hat{M}^{*a}	S^a	$\Delta^2 TP$
1952	2.0783	6.7045	3.5078	8.3695	...
1953	5.1637	6.5733	4.1100	4.5039	-0.20
1954	6.6527	7.2681	4.0211	4.3801	0.20
1955	9.1374	5.9758	5.3286	4.1665	0.10
1956	9.2250	6.3239	4.7189	4.3311	1.20
1957	6.2908	3.2286	4.2620	4.1723	2.40
1958	7.3733	-0.2535	3.0705	4.1107	-1.90
1959	2.9867	5.5611	5.2015	4.8654	-0.90
1960	10.0965	6.4030	3.6997	4.5022	0.20
1961	8.6522	3.5113	4.8176	4.6174	0.90
1962	4.5050	7.0558	7.3344	4.4632	-1.60
1963	8.7861	3.9827	6.9932	4.3639	1.80
1964	5.8134	8.5477	9.0243	4.6347	0.00
1965	8.9901	5.8238	7.8865	5.1194	0.20
1966	6.4425	9.2417	7.3344	5.3991	-0.30
1967	6.1259	5.4265	6.4116	5.6668	-0.40
1968	3.9213	7.2176	10.0856	6.1301	1.00
1969	11.3142	7.2564	8.3849	6.2924	-2.40
1970	8.1204	8.1057	7.3923	6.5517	2.90
1971	7.6057	8.5516	10.2734	6.7540	-0.40
1972	6.4219	12.8741	13.3905	6.9199	-0.80
1973	8.1501	12.3851	18.1196	6.9752	-0.40
1974	8.3081	7.1692	16.4725	6.5256	2.10
1975	3.6401	5.3633	7.3713	7.0515	-1.30
Mean Values over the Period 1954-75					
	7.2073	6.6827	7.7998	5.3634	...

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